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Summary

Ultrahigh energy aspects of perturbative string scattering are surveyed after our precedent compendia on the thermal Virasoro amplitude as well as the thermal Veneziano amplitude in proper reference to the possible violation of unitarity bounds. The configuration of highly-excited string states is then touched upon from the viewpoint of string cosmology in association with the possible macroscopic nonlocality as well as the newfashioned percolation scenario.

Elaboration of thermal string theories based upon the thermofield dynamics (TFD) [1] deserves more than passing consideration in the thermodynamical investigation of the thermal string ensemble in general. In a precedent compendium of ours [2], the thermal stability of planar duality was recapitulated after our previous publications on the TFD algorithm of the thermal Lovelace-Veneziano formula. In a sequent compendium of ours [3], the thermal stability of non-planar duality was epitomized after our previous publication on the TFD paradigm of the thermal Virasoro formula. The principal conclusion is summarized as follows: The thermal stability of the non-planar, four-tachyon tree amplitude is substantiated in the sense of the Virasoro formula [4,5] with the aid of the topological sewing machinery just through the thermal stability of the factorized, planar, four-tachyon tree amplitudes in the sense of the Veneziano formulae [6]. In another compendium of ours [7], the microcanonical ensemble paradigm of black hole thermodynamics was encapsulated after our previous publications on primordial black holes of various geometries. The principal observation is as follows: The most probable microcanonical distribution of black holes is self-consistently described at high energies in asymptotically flat space through the so-called single-massive-mode dominance scenario which is reminiscent of the newfashioned percolation scenario of the black hole ensemble. In the present communication, ultrahigh energy aspects of perturbative string scattering are surveyed on the basis of the thermal Virasoro formula [3] as well as the thermal Veneziano formula [2] in proper respect of the possible violation of unitarity bounds such as the Froissart bound [8] and the Cerulus-Martin bound [9]. The configuration of highly-excited string states is then sketched after ref. [3] and ref. [7] from the standpoint of string cosmology [10-15] with regard to the possible macroscopic nonlocality due to string extendedness and/or strong gravitational effects. The possible association with the newfashioned percolation scenario of the black hole ensemble is also touched upon.

Let us start with illustrating after ref. [3] the non-planar, four-tachyon tree amplitude of closed bosonic thermal strings in proper reference to the newfashioned Sommerfeld-Watson (SW) transform with the aid of the TFD thermal propagator [16] in the standard dispersion theoretic approach based upon the TFD calculus. The non-planar, four-tachyon tree amplitude $V_{cl}(s,t,u)$ of closed bosonic strings is obtained at zero temperature as a simple and natural consequence of sewing up planar, four-tachyon tree amplitudes of open bosonic strings. We are then led to the Virasoro amplitude [4,5]:

$$
\begin{aligned}
V_{cl}(s,t,u) &= g^2 \frac{\Gamma(-\frac{1}{2}\alpha_{cl}(s))\Gamma(-\frac{1}{2}\alpha_{cl}(t))\Gamma(-\frac{1}{2}\alpha_{cl}(u))}{\Gamma(-\frac{1}{2}[\alpha_{cl}(s)+\alpha_{cl}(t)])\Gamma(-\frac{1}{2}[\alpha_{cl}(t)+\alpha_{cl}(u)])\Gamma(-\frac{1}{2}[\alpha_{cl}(u)+\alpha_{cl}(s)])} \\
&= g^2 \frac{\Gamma(-1-\frac{s}{8})\Gamma(-1-\frac{t}{8})\Gamma(-1-\frac{u}{8})}{\Gamma(2+\frac{s}{8})\Gamma(2+\frac{t}{8})\Gamma(2+\frac{u}{8})}, \qquad (1)
\end{aligned}
$$

where

$$
\alpha(\zeta) \equiv \alpha_{cl}(\zeta) = 2\alpha_{op}(\frac{\zeta}{4}) = 2\alpha_{op} + \frac{1}{2}\alpha'_{op}\cdot\zeta = \alpha_{cl} + \alpha'_{cl}\cdot\zeta = 2 + \frac{\zeta}{4}, \qquad (2)
$$

Γ reads the gamma function, and g is the coupling constant of the closed bosonic string. In addition, the tachyon trajectory function $\alpha_{cl}(\zeta)$ [$\alpha_{op}(\zeta)$] of closed [open] bosonic string satisfies the constraint

$$
\alpha_{cl}(s) + \alpha_{cl}(t) + \alpha_{cl}(u) = -2 \quad [\alpha_{op}(s) + \alpha_{op}(t) + \alpha_{op}(u) = -1]. \qquad (3)
$$

Our task is then reduced to building up the non-planar, four-tachyon tree amplitude $V^{\beta}_{cl}(s,t,u)$ of closed bosonic strings at nonzero temperature in the dispersion theoretic TFD approach. Since there is no preferred channel for thermal fluctuations, the newfashioned SW transform of the non-planar thermal amplitude $V^{\beta}_{cl}(s,t,u)$ turns out to be

$$
\begin{aligned}
V^{\beta}_{cl}(s,t,u) = \frac{g^2}{3}\left(\frac{i}{2\pi}\right)^2 \int_{L-i\infty}^{L+i\infty} & dl_s dl_t \frac{\bar{V}^{\beta}(l_s,\frac{1}{2}\alpha_{cl}(s))\bar{V}^{\beta}(l_t,\frac{1}{2}\alpha_{cl}(t))}{\Gamma(-l_s-l_t)\Gamma(1+l_s+l_t)} \\
\times & \frac{\Gamma(-l_s)\Gamma(-\frac{1}{2}\alpha_{cl}(u))}{\Gamma(-l_s-\frac{1}{2}\alpha_{cl}(u))}\frac{\Gamma(-l_t)\Gamma(-\frac{1}{2}\alpha_{cl}(u))}{\Gamma(-l_t-\frac{1}{2}\alpha_{cl}(u))} \\
+ & \{s\to t;\ t\to u;\ u\to s\} + \{s\to u;\ t\to s;\ u\to t\}, \qquad (4)
\end{aligned}
$$

where $\beta = 1/kT$. The thermal, partial-fraction amplitude $\bar{V}^{\beta}(l_\zeta, \alpha_{cl}(\zeta)/2)$ is expressed in the TFD fashion as [16]

$$
\bar{V}^{\beta}(l_\zeta, \frac{1}{2}\alpha_{cl}(\zeta)) = -\left\{\frac{1}{l_\zeta - \frac{1}{2}\alpha_{cl}(\zeta)} + \frac{2\pi i}{e^{\beta\omega_\zeta/2}-1}\delta(l_\zeta - \frac{1}{2}\alpha_{cl}(\zeta))\right\}, \qquad (5)
$$

where $\omega_\zeta = \sqrt{|\zeta|}$; $\zeta = s,t,u$. Insertion of eq. (5) into eq. (4) yields the thermal Virasoro amplitude:

$$V_{cl}^{\beta}(s,t,u) = \frac{g^2}{3}\left\{\frac{e^{\beta\omega_s/2}}{e^{\beta\omega_s/2}-1}\frac{e^{\beta\omega_t/2}}{e^{\beta\omega_t/2}-1}+\frac{e^{\beta\omega_t/2}}{e^{\beta\omega_t/2}-1}\frac{e^{\beta\omega_u/2}}{e^{\beta\omega_u/2}-1}+\frac{e^{\beta\omega_u/2}}{e^{\beta\omega_u/2}-1}\frac{e^{\beta\omega_s/2}}{e^{\beta\omega_s/2}-1}\right\} \times \frac{\Gamma(-1-\frac{s}{8})\Gamma(-1-\frac{t}{8})\Gamma(-1-\frac{u}{8})}{\Gamma(2+\frac{s}{8})\Gamma(2+\frac{t}{8})\Gamma(2+\frac{u}{8})} \qquad (6)$$

in the tree approximation. Thus the topological sewing machinery *à la* non-planar fashion yields no disturbance of the thermal stability of the factorized, planar, four-tachyon tree amplitudes of open bosonic thermal strings. We are then led to conclude that the thermal stability of non-planar duality is substantialized at least at the tree level in the sense of the Virasoro formula in full consonance with the thermal stability of planar duality in the sense of the Veneziano formula. At sufficiently low temperatures, indeed, the thermal Virasoro amplitude (6) eventuates in the zero-temperature Virasoro amplitude (1). At very high temperatures, on the other hand, we obtain asymptotically

$$V_{cl}^{\beta}(s,t,u) \simeq g_{\mathit{eff}}^2 \frac{\Gamma(-1-\frac{s}{8})\Gamma(-1-\frac{t}{8})\Gamma(-1-\frac{u}{8})}{\Gamma(2+\frac{s}{8})\Gamma(2+\frac{t}{8})\Gamma(2+\frac{u}{8})} \;;\quad \beta\sim 0, \qquad (7)$$

where the effective coupling constant g_{eff} of the closed bosonic thermal string is expressed as

$$g_{\mathit{eff}}^2 = g^2(kT)^2\cdot\frac{4}{3}\frac{\omega_s+\omega_t+\omega_u}{\omega_s\omega_t\omega_u} \qquad (8)$$

at nonzero, finite values of s, t and u. The asymptotic expression (8) is reminiscent of the Atick-Witten formula [17], $g_{\mathit{eff}}^2 \sim g^2(kT)^2$, for the effective theory of closed bosonic strings at extremely high temperatures.

Let us now touch upon after our previous publications [18] the planar, four-tachyon tree amplitude $V_{op}^{\beta}(s,t,u)$ of open bosonic strings at nonzero temperature. The zero-temperature, planar, four-tachyon tree amplitude $V_{op}(s,t,u)$ is written in the form

$$V_{op}(s,t,u) = \bar{g}^2\left\{B(-\alpha_{op}(s),-\alpha_{op}(t))+B(-\alpha_{op}(t),-\alpha_{op}(u))+B(-\alpha_{op}(u),-\alpha_{op}(s))\right\}, \qquad (9)$$

where B reads the Euler beta function, and $\bar{g}$ is the coupling constant of the open bosonic string. Here, it is noted that $\bar{g}^2 \sim g$ as a simple and natural consequence of the topological sewing machinery. Accordingly, the planar, four-tachyon thermal amplitude $V_{op}^{\beta}(s,t,u)$ is described *à la* ref. [2] as

$$\begin{aligned} V_{op}^{\beta}(s,t,u) =& \frac{\bar{g}^2}{2}\left(\frac{e^{\beta\omega_s}}{e^{\beta\omega_s}-1}+\frac{e^{\beta\omega_t}}{e^{\beta\omega_t}-1}\right)B(-1-\frac{s}{2},-1-\frac{t}{2}) \\ &+\frac{\bar{g}^2}{2}\left(\frac{e^{\beta\omega_t}}{e^{\beta\omega_t}-1}+\frac{e^{\beta\omega_u}}{e^{\beta\omega_u}-1}\right)B(-1-\frac{t}{2},-1-\frac{u}{2}) \\ &+\frac{\bar{g}^2}{2}\left(\frac{e^{\beta\omega_u}}{e^{\beta\omega_u}-1}+\frac{e^{\beta\omega_s}}{e^{\beta\omega_s}-1}\right)B(-1-\frac{u}{2},-1-\frac{s}{2}) \end{aligned} \qquad (10)$$

in the tree approximation, where use has been made of the newfashioned SW transform:

$$V_{op}^{\beta}(s,t,u) = \frac{\bar{g}^2}{2}\frac{i}{2\pi}\int_{L-i\infty}^{L+i\infty}\left\{dl_s \bar{V}^{\beta}(l_s,\alpha_{op}(t))\frac{\Gamma(-l_s)\Gamma(-\alpha_{op}(t))}{\Gamma(-l_s-\alpha_{op}(t))} + (s\leftrightarrow t)\right\}$$
$$+\{s\to t;\ t\to u;\ u\to s\} + \{s\to u;\ t\to s;\ u\to t\} \tag{11}$$

with the aid of the TFD thermal propagator [16]:

$$\bar{V}^{\beta}(l_\zeta,\alpha_{op}(\zeta)) = -\left\{\frac{1}{l_\zeta-\alpha_{op}(\zeta)} + \frac{2\pi i}{e^{\beta\omega_\zeta}-1}\delta(l_\zeta-\alpha_{op}(\zeta))\right\}. \tag{12}$$

Thus the thermal stability of planar duality is guaranteed in the original sense of the Veneziano formula. At the low temperature limit, indeed, the thermal Veneziano amplitude (10) is evidently reduced to the zero-temperature Veneziano amplitude (9). At the high temperature limit, on the other hand, we obtain asymptotically

$$V_{op}^{\beta}(s,t,u) \simeq \bar{g}^2_{st;eff}B(-1-\frac{s}{2},-1-\frac{t}{2}) + \bar{g}^2_{tu;eff}B(-1-\frac{t}{2},-1-\frac{u}{2})$$
$$+\bar{g}^2_{us;eff}B(-1-\frac{u}{2},-1-\frac{s}{2})\ ;\quad \beta\sim 0, \tag{13}$$

where

$$\bar{g}^2_{\zeta_1\zeta_2;eff} = \bar{g}^2 kT\cdot\frac{1}{2}\frac{\omega_{\zeta_1}+\omega_{\zeta_2}}{\omega_{\zeta_1}\omega_{\zeta_2}}\ ;\quad \zeta_1,\zeta_2 = s,t,u \tag{14}$$

at nonzero, finite values of s, t and u. The asymptotic relation (14) is reminiscent of the Atick-Witten formula [17], $\bar{g}^2_{eff}\sim\bar{g}^2 kT$, for the effective theory of open bosonic strings at very high temperatures.

Let us turn our attention to high energy aspects of four-tachyon tree amplitudes $V_{cl}^{\beta}(s,t,u)$ and $V_{op}^{\beta}(s,t,u)$ of thermal bosonic strings. The high energy, fixed-angle behaviour of the thermal Virasoro amplitude (6) is reduced to

$$V_{cl}^{\beta}(s,t,u) \sim V_{cl}(s,t,u)$$
$$\sim 8ig^2e^{-8}(stu)^{-3}\exp\left[-\frac{1}{4}(s\ln s + t\ln t + u\ln u)\right] \tag{15}$$

at any finite temperature, where use has been made of the Stirling formula. Irrespective of the thermal disturbance, therefore, the asymptotic behaviour (15) is identical with the observation of Gross and Mende [19] on the zero-temperature, four-tachyon tree amplitude of closed bosonic strings. Similarly, the high energy, fixed-angle behaviour of the thermal Veneziano amplitude (10) turns into

$$V_{op}^{\beta}(s,t,u) \sim V_{op}(s,t,u)$$
$$\sim -2\bar{g}^2e^{-4}(stu)^{-3/2}\exp\left[-\frac{1}{2}(s\ln s + t\ln t + u\ln u)\right] \tag{16}$$

at any finite temperature, irrespective of the thermal disturbance, which is identical with the original observation on the zero-temperature, four-point tree amplitude of open bosonic strings by Veneziano [6,20]. Asymptotic expressions (15) and (16) violate the Cerulus-Martin lower bound on the high-energy, fixed-angle amplitude which reads [9,21]

$$|F(s, \cos\theta)| \geq \exp[-f(\theta)\sqrt{s}\ln s] \tag{17}$$

with some appropriate function $f(\theta)$. The derivation of the lower bound (17) is accomplished for the polynomial boundedness condition in the Mandelstam representation, but nevertheless inapplicable in the presence of infinitely-rising Regge trajectories such as eq. (2). Accordingly, the asymptotic behaviours (15) and (16) might cause no conceptual inconsistency to any physical principle, *e.g.* unitarity.

The high-energy, fixed-momentum-transfer behaviour of the thermal Virasoro fourmula (6) is written in the standard Regge formalism as

$$V_{cl}^{\beta}(s,t,u) = g_R^2/g^2 \cdot V_{cl}^R(s,t) \tag{18}$$

at any nonzero, finite temperature, where

$$\begin{aligned} V_{cl}^R(s,t) &\sim \pi g^2 \frac{e^{-2-t/4}}{[\Gamma(2+t/8)]^2}\left(\frac{s}{8}\right)^{2+t/4}\left\{i - \cot\left(\frac{\pi t}{8}\right)\right\} \\ &\to \pi g^2 l_s^4 e^{-2}\left(\frac{s}{8}\right)^2\left\{i - \frac{8}{\pi t}\right\} \;; \quad t \sim 0 \end{aligned} \tag{19}$$

and

$$g_R^2 \sim \begin{cases} g^2 \dfrac{2}{3}\dfrac{e^{\beta\omega_t/2}}{e^{\beta\omega_t/2}-1} \;; & 2/\omega_s \lesssim \beta < \infty \\ g^2 (kT)^2 \dfrac{8}{3}\dfrac{1}{\omega_s\omega_t} \;; & 0 < \beta < 2/\omega_s \end{cases} \tag{20}$$

which is asymptotically reduced to

$$g_R^2 \sim g^2 \omega_s l_s \;; \quad s \to \infty \;; \quad t \sim 0. \tag{21}$$

Here, $V_{cl}^R(s,t)$ is the Regge asymptotic form of the zero-temperature Virasoro formula (1) and the fundamental string length $l_s \sim \sqrt{\alpha'}$ is in association with the string tension which reads $(2\pi\alpha')^{-1}$. Thus the thermal Virasoro amplitude (6) yields the total cross section of the form

$$\sigma_{cl}^T(s) \simeq \frac{1}{s}\mathrm{Im}V_{cl}^{\beta}(s, t \simeq 0, u) \sim \pi g_R^2 l_s^4 s \;; \quad s \to \infty \tag{22}$$

up to a numerical factor in the tree approximation. The asymptotic expression (19) violates the Froissart upper bound on the high-energy, forward amplitude which reads [8,21]

$$F(s, t \simeq 0) \lesssim Cs(\log s)^2 \tag{23}$$

with some appropriate constant C. The derivation of the upper bound (23) is established for local quantum field theory as an inevitable consequence of unitarity in the sense of the so-called small Lehmann-Martin ellipse [21] with regard to the nearest, nonvanishing mass, but nevertheless inapplicable for closed string theory in association with massless modes in the sense of the absence of a gap. Accordingly, the asymptotic behaviour (19) might cause no conceptual contradiction to any physical principle, *e.g.* unitarity. It is parenthetically mentioned that the cross section $\sigma_{cl}^T(s)$ saturates the Froissant bound

$$\sigma_{cl}^T(s) \sim \pi l_s^2 \tag{24}$$

up to a logarithmic factor at around $\omega_s \sim (g^{2/3} l_s)^{-1}$. The increasing cross section (22) is heuristically considered as arising from production of highly excited, stretched strings of length $\omega_s l_s^2$ at high energies in the sense of string uncertainty principle *à la* Veneziano [22] which reads

$$\Delta x \gtrsim 1/\Delta p + l_s^2 \Delta p. \tag{25}$$

The production amplitude of a long, highly excited, closed string state as a massive resonance in the s-channel is, indeed, dual at sufficiently high energies to the long-distance exchange amplitude of a short, light or equivalently massless, closed string state in the t-channel, corresponding to the single graviton exchange over a large distance. As already argued by Emparan *el al.* [23,24], consequently, the production cross section of a highly massive, closed string state at mass level ω_s is asymptotically described as eq. (22). Let us tentatively postulate validity of the Froissart bound (23) in string theory without loss of generality. It will then be possible to claim *à la* ref. [23] at least at sufficiently small coupling g that the production cross section (22) mentioned above grows with s for $l_s^{-1} \ll \omega_s < (g^{2/3} l_s)^{-1} \sim g^{4/3}\omega_c$, while remains constant at the saturated value (24) for $g^{4/3}\omega_c \lesssim \omega_s \lesssim \omega_c \sim (g^2 l_s)^{-1}$, where ω_c reads the mass level at the string/black hole correspondence point. Here, the Hagedorn temperature ω_c/k is equal to the maximal value of the Hawking temperature for the correspondent black hole in association with the minimal value l_s of the Schwarzschild radius. Let us call to remembrance that $\sigma_{cl}^T(s)$ can be identified at $\omega_s \sim \omega_c$ with the production cross section of a black hole of the Schwarzschild radius l_s. The detailed discussion on the string/black hole correspondence as well as the total cross section $\sigma_{cl}^T(s)$ for the case $\omega_s \gtrsim \omega_c \sim (g^2 l_s)^{-1}$ is referred to the last paragraph.

Similarly, the Regge behaviour of the thermal Veneziano formula (10) is expressed as

$$V_{op}^{\beta}(s,t,u) = \bar{g}_R^2/\bar{g}^2 \cdot V_{op}^R(s,t) \tag{26}$$

at any nonzero, finite temperature, where

$$V_{op}^{R}(s,t) \sim \pi \bar{g}^2 \frac{e^{-1-t/2}}{\Gamma(2+t/2)} \left(\frac{s}{2}\right)^{1+t/2} \left\{ i + \tan\left(\frac{\pi t}{4}\right) \right\}$$
$$\to \pi \bar{g}^2 l_s^2 e^{-1} \frac{s}{2} \left\{ i + \frac{\pi t}{4} \right\} \ ; \quad t \sim 0 \tag{27}$$

and

$$\bar{g}_R^2 \sim \begin{cases} \bar{g}^2 \dfrac{1}{2} \dfrac{e^{\beta\omega_t}}{e^{\beta\omega_t}-1} \ ; & 1/\omega_s \lesssim \beta < \infty \\ \bar{g}^2 kT \dfrac{1}{2} \dfrac{1}{\omega_t} \ ; & 0 < \beta < 1/\omega_s \end{cases} \tag{28}$$

which is asymptotically reduced to

$$\bar{g}_R^2 \sim \bar{g}^2 \omega_s l_s \ ; \quad s \to \infty \ ; \quad t \sim 0. \tag{29}$$

Here, $V_{op}^{R}(s,t)$ is the Regge limit of the zero-temperature Veneziano formula (9). Thus the thermal Veneziano amplitude (10) brings forth the total cross section of the form

$$\sigma_{op}^{T}(s) \simeq \frac{1}{s} \mathrm{Im} V_{op}^{\beta}(s, t \simeq 0, u) \sim \pi \bar{g}_R^2 l_s^2 \ ; \quad s \to \infty \tag{30}$$

up to a numerical factor in the tree approximation. The asymptotic expression (27) saturates the Froissart bound (23) up to a logarithmic factor. The production cross section of a long, highly excited, open string state at mass level ω_s is asymptotically described as eq. (30) in association with the exchange of a short, light, open string state instead of the single graviton exchange. It seems that the open string cross section (30) is subdominant to the closed string cross section (22) at sufficiently high energies such as $\omega_s > (g^{1/2} l_s)^{-1} \sim g^{3/2}\omega_c$ in proper reference to the production mechanism of a highly massive, string state. It may be stated parenthetically that the cross section $\sigma_{op}^{T}(s)$ turns out to be

$$\sigma_{op}^{T}(s) \sim \pi g^{1/3} l_s^2; \quad g < 1 \tag{31}$$

at $\omega_s \sim (\bar{g}^{4/3} l_s)^{-1} \sim g^{4/3}\omega_c$ and saturates the Froissart bound

$$\sigma_{op}^{T}(s) \sim \pi l_s^2 \tag{32}$$

up to a logarithmic factor at around $\omega_s \sim (\bar{g}^2 l_s)^{-1} \sim g\omega_c$, which is identical with eq. (24) and turns out to be equal in magnitude to the production cross section of a black hole of the Schwarzschild radius l_s. Let us tentatively suppose applicability of the Froissart bound (23) to string theory in general. It will then be possible to argue at least at sufficiently small coupling g that the production cross section (30) grows with s for $l_s^{-1} \ll \omega_s < (\bar{g}^2 l_s)^{-1} \sim g\omega_c$, while remains constant at the saturated value (32) for $g\omega_c \lesssim \omega_s \lesssim \omega_c \sim (g^2 l_s)^{-1}$, where $g\omega_c$ is of the same order as the Planck mass scale. As compared with the argument on the cross section $\sigma_{cl}^{T}(s)$, the observation mentioned above will imply that $\sigma_{op}^{T}(s)/\sigma_{cl}^{T}(s) \sim (g\omega_s^2 l_s^2)^{-1} < 1$ for $g^{3/2}\omega_c < \omega_s < g^{4/3}\omega_c$, $\sigma_{op}^{T}(s)/\sigma_{cl}^{T}(s) \sim g\omega_s l_s < 1$ for $g^{4/3}\omega_c \lesssim \omega_s < g\omega_c$, and $\sigma_{op}^{T}(s)/\sigma_{cl}^{T}(s) \sim 1$ for $g\omega_c \lesssim \omega_s \lesssim \omega_c \sim (g^2 l_s)^{-1}$. Accordingly, $\sigma_{op}^{T}(s)$ will be literally subleading with regard to

$\sigma^T_{cl}(s)$ at $g^{3/2}\omega_c < \omega_s < g\omega_c$, while the open string, thermal tachyon may play the same role as the closed string, thermal tachyon in respect of the production mechanism of a highly massive, string state beyond the Planck mass scale $(gl_s)^{-1}$, in consonance with the possible unification of all forces in nature, *i.e.* strong, electromagnetic, weak and gravitational interactions. The detailed behaviour of $\sigma^T_{op}(s)/\sigma^T_{cl}(s)$ might still remain unsettled at ultrahigh energies $\omega_s \gtrsim g^{4/3}\omega_c$, however.

By way of illustration, the configuration of highly-excited, closed bosonic thermal strings is sketched after ref. [3] as well as ref. [7] at trans-Planckian energies, *i.e.*

$$\omega_s > M_s = 1/l_s \sim 1/\sqrt{\alpha'} \ ;$$
$$\omega_s > M_{\rm P} = 1/l_{\rm P} = 1/\sqrt{G} \sim 1/gl_s, \tag{33}$$

from the viewpoint of string cosmology [10-15], where $M_{\rm P}$ and $l_{\rm P}$ are Planck mass and Planck length, respectively. It is of interest to note that $M_s \sim g^2\omega_c$, $M_{\rm P} \sim g\omega_c$ and $l_{\rm P} \sim gl_s$. Here, the sufficiently small string coupling

$$g \sim l_{\rm P}/l_s = M_s/M_{\rm P} < 1 \tag{34}$$

has been postulated in perturbative string scattering and use is made of $\alpha'_{cl} = 1/2 \cdot \alpha'_{op} \equiv \alpha' \neq 1/4$ and $G \neq 1$ besides $c = \hbar = k = 1$ in the present context. The paradigm of string cosmology is based upon the hypothesized holographic principle and the conjectured correspondence principle. In particular, the degree of freedom inside a black hole, *i.e.* the number of microscopic states in relation to a black hole, is described by the Bekenstein-Hawking entropy which is not proportional to its volume but proportional to its horizon area and eventually turns out to be the logarithmic mass degeneracy of the correspondent self-gravitating string state at least for the case of a nearly extremal black hole. The argument that black holes form in ultrahigh energy collision of two localized objects rests on semiclassical algorithm in local quantum field theory in the sense of the approximation in which one string scatters in the approximate Aichelburg-Sexl metric of the other string [25-30]. In addition, strong gravitational dynamics may lead to the possible failure of local quantum field theory on scales much larger than the Planck length $l_{\rm P}$ at ultrahigh energies when a given energy ω_s is concentrated inside a closed trapped domain, *i.e.* a black hole, of the Schwarzschild radius $R_{\rm S} \sim \omega_s g^2 l_s^2 \sim (\omega_s/\omega_c)l_s$. On the other hand, a highly excited string of energy ω_s can stretch over a distance $\omega_s l_s^2$ and might yield macroscopic nonlocality much larger than the string scale l_s at ultrahigh energies. Such stringy nonlocality could prevent formation of black holes in high energy collisions, because the string energy distribution spreads out on scales $\omega_s l_s^2$ large as conpared to the supposed horizon of the Schwarzschild radius $R_{\rm S} \sim \omega_s g^2 l_s^2 \sim (\omega_s/\omega_c)l_s$. There is no manifest indication for such long-string effects, however. It is once again reminded that creation of a long, highly-excited, string state in the s-channel is dual at sufficiently high energies to long-distance exchange of a short, light, string state of the graviton mode in the t-channel. The string interaction producing the exchange of the graviton mode is then nonlocal on the microscopic scale l_s but not on the macroscopic scale such as $\omega_s l_s^2$ at ultrahigh energies. The scattering amplitude is naively dominated by the long-range gravity beyond a scale of tidal string excitation which reads [26,29,30] $l_D \sim \omega_s g l_s^2$. Significant tidal excitation might cause some sort of stringy nonlocality. Moreover, the tidal excitation scale l_D is larger than the supposed Schwarzschild radius $R_{\rm S}$. As has often been claimed by Giddings *et al.* [29,30], however, such tidal excitation effects can not interfere with formation of black holes in ultrahigh energy collisions, because colliding strings of size l_s is concentrated inside a closed trapped region of size $R_{\rm S}$ at ultrahigh energies $\omega_s > \omega_c \sim (g^2 l_s)^{-1}$ by the time one string experiences a tidal force in the gravitational field of the other string. Thus there exists no manifest indication for such stringy nonlocality due to tidal excitation. Accordingly, extendedness of the string will not cause long-distance nonlocal effects which interfere with formation of a closed trapped surface at ultrahigh energies $\omega_s > \omega_c \sim (g^2 l_s)^{-1}$, at which the Schwarzschild radius $R_{\rm S}$ exceeds the string length l_s. We are then led *à la* [29,30] to principal conclusions: Firstly, there will be no adduction indicating macroscopic nonlocality intrinsic to extendedness of the string. Secondly, black hole formation will be inherent in strong gravitational dynamics without interference due to stringy nonlocality. Thirdly, the possible breakdown of locality at scales $R_{\rm S}$ will be inevitably associated with breakdown of gravitational perturbation theory at the black hole threshold. Accordingly, nonperturbative gravitational physics will be essentially nonlocal. Finally, violation of asymptotic bounds for local field theory, such as the Froissart upper bound, may be intimately connected with macroscopic nonlocality intrinsic to gravitational nonperturbative dynamics.

In the investigation of the string/black hole correspondence, by way of parenthesis, it is of practical importance to note a paradox that the typical scale of a highly-excited string at mass level ω_s will be described in the absence of gravitational self-interaction as the radius of a random walk composed of $\omega_s l_s$ steps and length l_s which reads [11,13,23,31] $R_{rw} \sim \omega_s^{1/2} l_s^{3/2}$ at high energies in accordance with the corresponding string entropy $S_s \sim \omega_s l_s$. The random walk radius R_{rw} can never contract down to the string scale at $\omega_s > M_s$. As already argued by Veneziano *et al.* [31], however, the pathological situation mentioned above will be remedied in the microcanonical ensemble paradigm of self-gravitating strings. As a consequence of the competition between the centrifugal barrier and the gravitational potential, indeed, the size of a typical self-gravitating string is determined by [31] $R_{typ} \sim 1/g^2\omega_s$; $(\omega_s l_s)^{-3/2} \lesssim g^2 \lesssim (\omega_s l_s)^{-1}$, which correctly interpolates between the larger random walk size R_{rw} and the compact string size l_s. As has often been emphasized by ourselves [32,33], there appears the maximum temperature $T_c \sim M_s$ of string excitation, irrespective of the detailed structure of the thermal string ensemble, beyond which the thermal string amplitude is infrared divergent. The maximum temperature T_c is of the same order as the Hagedorn temperature $\hat{T}_{\rm H} \sim M_s$ of the thermal string ensemble, beyond which the canonical partition function diverges for sufficiently large values of mass. It may be stated parenthetically that $\hat{T}_{\rm H} = M_s/4\pi$ for the bosonic thermal string, $\hat{T}_{\rm H} = M_s/2\sqrt{2}\pi$ for the type II thermal string and $\hat{T}_{\rm H} = M_s/(2+\sqrt{2})\pi$ for the heterotic thermal string. The critical temperature $T_c \sim M_s$ seems to be interpreted as the phase transition temperature from the overall standpoint of string thermodynamics. The string/black hole correspondence is then recapitulated as follows: The string entropy $S_s \sim \omega_s l_s$ at mass level ω_s turns out to be the same order as the Bekenstein-Hawking entropy $S_{\rm BH} \sim \omega_s^2 g^2 l_s^2 \sim (\omega_s/\omega_c) S_s$ of the corresponding black hole when the string length l_s becomes of the order of the Schwarzschild radius $R_{\rm S} \sim \omega_s g^2 l_s^2 \sim (\omega_s/\omega_c) l_s$. The effective coupling constant squared $g_{\it eff}^2$ is asymptotically reduced to $g_{\it eff}^2 \sim g^2 \omega_s l_s \sim \omega_s/\omega_c$ for mass level $\omega_s \gg M_s$. If $g^2 < (\omega_s l_s)^{-1}$, *i.e.* $\omega_s < \omega_c$, then, the Hawking temperature $T_{\rm H} \sim (\partial S_{\rm BH}/\partial \omega_s)^{-1} \sim (\omega_s g^2 l_s^2)^{-1} \sim R_{\rm S}^{-1} \sim (\omega_c/\omega_s) M_s$ is higher than the Hagedorn temperature $\hat{T}_{\rm H} \sim M_s$ and the string will spread out on scales much larger than the supposed horizon so that the black hole is depicted as a continuum string state. If $g^2 > (\omega_s l_s)^{-1}$, *i.e.* $\omega_s > \omega_c$, on the other hand, the Hawking temperature $T_{\rm H}$ is lower than the Hagedorn temperature $\hat{T}_{\rm H}$ and the horizon will be much bigger than the string length scale so that the string energy is concentrated inside a closed trapped domain and consequently the string behaves as a black hole. Accordingly, the criterion $g_{\it eff}^2 \sim 1$ will effectively describe the string coupling squared at the string $\rightleftarrows$ black hole transition point. Thus $g_{\it eff}^2$ plays the role of the order parameter at asymptotically high energies. As a consequence, the critical temperature T_c is naturally interpreted as the phase transition temperature at which the thermal string configuration turns into a localized black hole and *vice versa*. The production cross section σ_{BH}^T of a black hole at mass level ω_s is geometrically written in the form

$$\begin{aligned} \sigma_{BH}^T &\sim \pi R_{\rm S}^2 \\ &\sim \pi \omega_s^2 g^4 l_s^4 \sim \pi(\omega_s/\omega_c)^2 l_s^2 \ ; \quad \omega_s \gtrsim \omega_c \sim 1/g^2 l_s \end{aligned} \tag{35}$$

which turns out to be

$$\sigma_{BH}^T \sim \pi l_s^2 \tag{36}$$

at the string $\rightleftarrows$ black hole transition point:

$$g_R^2 \sim g_{\it eff}^2 \sim g^2 \omega_s l_s \sim \omega_s/\omega_c \sim 1. \tag{37}$$

It is of interest to note that the production cross section (36) is identical with eq. (24), *i.e.* the saturated production cross section of a highly-excited string state at mass level ω_s for $g^{4/3}\omega_c \lesssim \omega_s \lesssim \omega_c$, or equivalently $(\omega_s l_s)^{-3} \lesssim g^2 \lesssim (\omega_s l_s)^{-1}$, which is in turn equal to eq. (32) for $g\omega_c \lesssim \omega_s \lesssim \omega_c$, *i.e.* $(\omega_s l_s)^{-2} \lesssim g^2 \lesssim (\omega_s l_s)^{-1}$. As a salient feature of our argument in ref. [7], the most probable microcanonical distribution of primordial black holes is self-consistently described at ultrahigh energies in asymptotically flat space through the so-called single-massive-mode dominance scenario in the sense that most of the mass, most of the charge and most of the angular momentum of the whole system converge on a single energetic black hole. As combined with the present argument on the string/black hole correspondence, then, the critical temperature T_c is naively reminiscent of the newfashioned percolation temperature at which the multi-black hole ensemble coalesces into a single primordial black hole of the critical mass ω_c which eventually transmutes into a single primordial string mode of the same mass. The observation mentioned above will be of active interest in association with the argument *à la* Susskind *et al.* [15] that the Hagedorn temperature $\hat{T}_{\mathrm{H}}$ effectively describes the so-called percolation temperature for the multi-string distribution to coalesce into a single string state. It still remains to be clarified in a nonperturbative fashion, however, whether or not the critical temperature T_c truly prescribes the disintegration point for the enigmatic phase transition of the primordial black hole system, and whether or not the so-called percolation scenario of the Hagedorn transition near T_c is fully effectual in elaborating the possible linkage between self-gravitating single string states and multi-string states.

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